Optimisation des structures plastiques par programmation conique

Jérémy Bleyer, Leyla Mourad, Romain Mesnil, Karam Sab

Laboratoire Navier, ENPC, Univ Gustave Eiffel, CNRS

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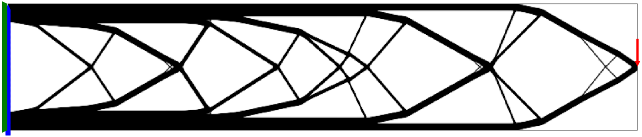
2 Juin 2022

Topology optimization : elastic setting

Find $\Omega \subseteq \mathcal{D}$ minimizing the **elastic compliance** at fixed volume [Allaire, 2002]:



[TopOpt in Python, DTU]

$$
\begin{aligned}
\min_{\Omega,\boldsymbol{u}} \quad & \int_{\partial\Omega} \boldsymbol{T}\cdot\boldsymbol{u}\,\mathrm{dS} \\
\text{s.t.} \quad & \boldsymbol{\sigma} = \mathbb{C}:\nabla\boldsymbol{u} \text{ in } \Omega \\
& \operatorname{div}\boldsymbol{\sigma} = 0 \quad \text{in } \Omega \\
& \boldsymbol{\sigma}\boldsymbol{n} = \boldsymbol{T} \quad \text{on } \partial\Omega_N \\
& |\Omega| \leq \eta|\mathcal{D}|
\end{aligned}
$$

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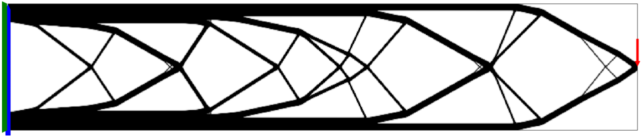

[TopOpt in Python, DTU]

Density-based formulation

$$\begin{array}{ll} \min\limits_{\rho,\boldsymbol{u}} & \displaystyle\int_{\partial\mathcal{D}} \boldsymbol{T}\cdot\boldsymbol{u}\,\mathrm{dS} \\ \text{s.t.} & \boldsymbol{\sigma} = \mathbb{C}(\rho) : \nabla\boldsymbol{u} \text{ in } \mathcal{D} \\ & \operatorname{div}\boldsymbol{\sigma} = 0 \quad \text{in } \mathcal{D} \\ & \boldsymbol{\sigma}\boldsymbol{n} = \boldsymbol{T} \quad \text{on } \partial\mathcal{D}_N \\ & \int_{\mathcal{D}} \rho\,\mathrm{d}\Omega \leq \eta|\mathcal{D}| \\ & 0 \leq \rho(\boldsymbol{x}) \leq 1 \end{array}$$

$\Rightarrow$ **non-convex** problem, iterative procedure

e.g. SIMP method [Bendsoe and Kikuchi, 1988] : $\mathbb{C}(\rho) = \rho^p\mathbb{C}_0$ with $p > 1$

Topology optimization : elastic setting

Limitations of compliance minimization with an **elastic behaviour**

- in practice, materials are non-linear (plasticity, tension/compression, etc.)
- does not account for stress limits
- not relevant for optimizing reinforcements e.g. steel in reinforced concrete

Extensions to **nonlinear behaviours**:

- local stress constraints make TopOpt formulations more difficult (non smoothness) [Duysinx and Bendoe, 1998]
- sensitivities (or shape derivatives) are often computed using regularized behaviours [Maury et al., 2018]
- need to optimize with respect to the whole loading path

Here, we want to optimize with the structure **limit load** rather than elastic compliance using **limit analysis theory**

Outline

Limit analysis theory: a convex optimization formulation

Collapse = there exist no **internal stress field** satisfying both **equilibrium** and **strength** conditions [Hill, 1950; Salençon, 1983]

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- **Stress field**: a symmetric 2nd-rank tensor $\boldsymbol{\sigma}(\boldsymbol{x})$ for 2D/3D solids
- **Equilibrium** with respect to a given loading :

$$\begin{aligned} \operatorname{div} \boldsymbol{\sigma} &= 0 \quad \text{for } \boldsymbol{x} \in \Omega \\ \boldsymbol{\sigma} \boldsymbol{n} &= \lambda \boldsymbol{T} \quad \text{for } \boldsymbol{x} \in \partial\Omega_N \end{aligned}$$

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- **Strength condition**: G **convex** set containing 0

$$\begin{aligned} & \boldsymbol{\sigma}(\boldsymbol{x}) \in G(\boldsymbol{x}) \quad \forall \boldsymbol{x} \in \Omega \\ \Leftrightarrow \quad & g_G(\boldsymbol{\sigma}) \leq 1 \end{aligned}$$

g_G Minkowski functional (gauge) of G

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Collapse load

Find the **maximum** load multiplier λ such that such a stress field exists

Convex optimization formulation

Continuous problem:

$$\lambda^+ = \max_{\lambda,\boldsymbol{\sigma}\in\mathcal{W}} \quad \lambda$$
$$\begin{array}{lll} & \operatorname{div}\boldsymbol{\sigma} = 0 & \forall \boldsymbol{x} \in \Omega \\ \text{s.t.} & \boldsymbol{\sigma}\boldsymbol{n} = \lambda \boldsymbol{T} & \forall \boldsymbol{x} \in \partial\Omega_N \\ & g_G(\boldsymbol{\sigma}) \leq 1 & \end{array}$$

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Discrete (e.g. finite-element) formulation:

$$\lambda^+ = \max_{\lambda, \boldsymbol{\sigma} \in \mathcal{W}_h} \quad \lambda$$
$$\text{s.t.} \quad \begin{array}{ll} \boldsymbol{H}\boldsymbol{\sigma} + \lambda \boldsymbol{F} = 0 & \\ g_G(\boldsymbol{\sigma}_k) \leq 1 & \forall k = 1, \dots, N \end{array}$$

convex optimization problems

Convex optimization formulation

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convex optimization problems

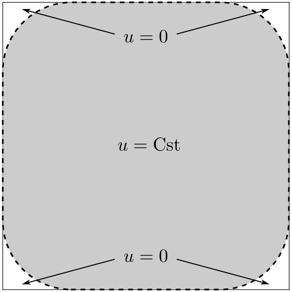
usually G (thus also g_G) has a simple geometrical shape: ellipsoid, cone, etc.

$\Rightarrow$ **conic programming solvers** e.g. `MOSEK`

Antiplane case : Cheeger problem [Cheeger, 1969]

Dual problem:

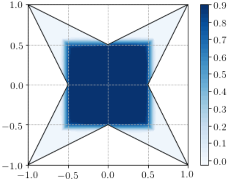
$$c_\Omega = \inf_{\omega\subseteq\Omega} \frac{|\partial\omega|}{|\omega|} = \begin{array}{rl} \displaystyle\inf_{u\in V_0} & \displaystyle\int_\Omega \|\nabla u\|_2 \,\mathrm{d}\Omega \\ \text{s.t.} & \displaystyle\int_\Omega fu \,\mathrm{d}\Omega = 1 \end{array}$$



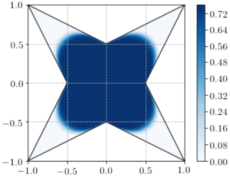
`fenics_optim` package: convex variational problems + finite-element discretization with `FEniCS` [Bleyer, 2020, ACM TOMS]

Antiplane case : Cheeger problem

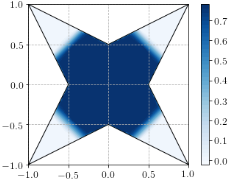
```
V = FunctionSpace(mesh, "CG", 2)
prob = MosekProblem("Cheeger problem")
u = prob.add_var(V, bc=bc)

F = L2Norm(grad(u), degree=0)
prob.add_convex_term(F)

f = Constant(1.)
R = FunctionSpace(mesh, "Real", 0)
def constraint(l):
    return l*f*u*dx
prob.add_eq_constraint(R, A=constraint, b=1)

prob.optimize()
```



(a) $\|\nabla u\|_1$

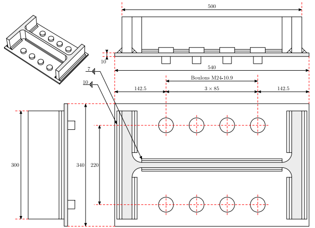


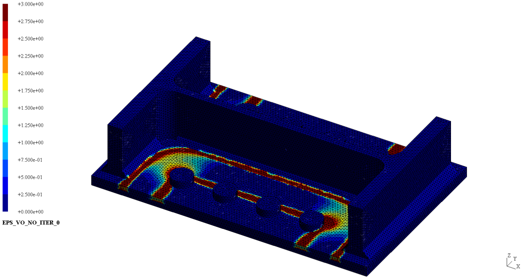
(b) $\|\nabla u\|_2$

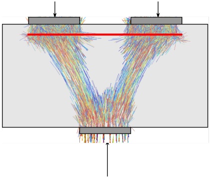


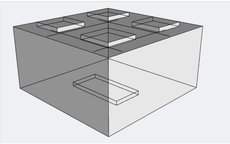
(c) $\|\nabla u\|_\infty$

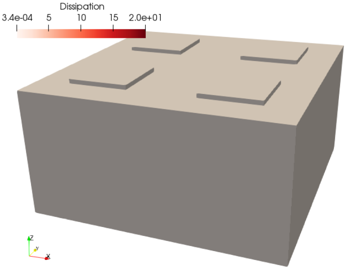
Examples


500
540
Boulons M24-10.9
142.5
3 × 85
142.5
10
7
10
300
340
220


+3.000e+00
+2.750e+00
+2.500e+00
+2.250e+00
+2.000e+00
+1.750e+00
+1.500e+00
+1.250e+00
+1.000e+00
+7.500e-01
+5.000e-01
+2.500e-01
+0.000e+00
EPS_VO_NO_ITER_0






Dissipation
3.4e-04
5
10
15
2.0e+01

Outline

Maximizing the limit load

Objective : Find $\Omega \subseteq \mathcal{D}$ with **maximum limit load** for a given volume level η:

Proposed formulation:

$$\begin{aligned} \lambda^+(\eta) = \max_{\lambda,\boldsymbol{\sigma},\Omega} \quad & \lambda \\ \text{s.t.} \quad & \operatorname{div} \boldsymbol{\sigma} = 0 && \text{in } \Omega \\ & \boldsymbol{\sigma}\boldsymbol{n} = \lambda \boldsymbol{T} && \text{in } \partial\Omega_N \\ & g_G(\boldsymbol{\sigma}) \leq 1 && \text{in } \Omega \\ & |\Omega| \leq \eta|\mathcal{D}| \end{aligned}$$

Maximizing the limit load

Objective : Find $\Omega \subseteq \mathcal{D}$ with **maximum limit load** for a given volume level η:

extension by $\boldsymbol{\sigma} = 0$ outside Ω

$$\begin{array}{rlll} \lambda^+(\eta) = \displaystyle\max_{\lambda,\boldsymbol{\sigma},\rho} & \lambda & \\ \text{s.t.} & \operatorname{div} \boldsymbol{\sigma} = 0 & \text{in } \mathcal{D} \\ & \boldsymbol{\sigma} \boldsymbol{n} = \lambda \boldsymbol{T} & \text{in } \partial\mathcal{D}_N \\ & g_G(\boldsymbol{\sigma}) \leq \rho & \text{in } \mathcal{D} \\ & \int_{\mathcal{D}} \rho \,\mathrm{d}\Omega \leq \eta|\mathcal{D}| & \\ & \rho \in \{0; 1\} & \end{array}$$

ρ being the characteristic function of Ω

Maximizing the limit load

Objective : Find $\Omega \subseteq \mathcal{D}$ with **maximum limit load** for a given volume level η:

problem **convexification** (LOAD-MAX)

$$
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Properties:

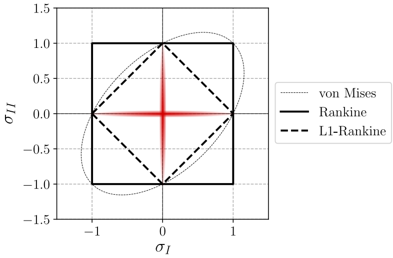
- $g_G(\boldsymbol{\sigma}) \leq \rho$ is a **convex constraint** in $(\boldsymbol{\sigma}, \rho)$
- akin to a limit analysis problem with an additional scalar variable ρ
- $\lambda^+(\eta = 1)$: limit load associated with $\mathcal{D}$

Choice of the strength criterion G

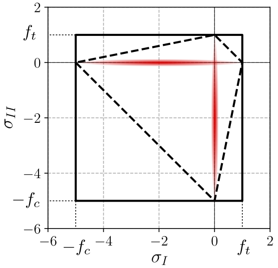
Optimized structures behave like trusses ($\boldsymbol{\sigma}$ uniaxial) $\boldsymbol{\sigma} \approx \begin{cases} +f_t \boldsymbol{e}_\alpha \otimes \boldsymbol{e}_\alpha \\ -f_c \boldsymbol{e}_\alpha \otimes \boldsymbol{e}_\alpha \end{cases}$

Proposition: L_1−Rankine criterion

$$g_{\text{L1-Rankine}}(\boldsymbol{\sigma}) = \sum_{J=I,II,III} \max\left\{-\frac{\sigma_J}{f_c}; \frac{\sigma_J}{f_t}\right\} \leq 1$$



(a) symmetric strengths $f_t = f_c$



(b) asymmetric strengths $f_c/f_t = 5$

Numerical implementation

Penalisation procedure

- **continuation** on $g_G(\boldsymbol{\sigma}) \leq \rho^p$ with $p \to p_{\max} > 1$ (SIMP-like)
  $\Rightarrow \rho$ is driven towards 0 or 1 (**truss**-like topology)
  At step n, linearisation around ρ_{n-1}:

$$\rho^{p_n} \approx \rho_{n-1}^{p_n} + p_n \rho_{n-1}^{p_n-1}(\rho - \rho_{n-1}) = a_n + b_n \rho$$

  typically $p_{\max} = 3$, 20 iterations

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- **localisation limiter** by gradient-control constraint

$$\|\nabla \rho\|_2 \leq 1/\ell$$

  with ℓ a regularization length (a few times larger than h) $\Rightarrow$ **mesh-independency**

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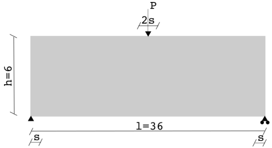
Finite-elements : `FEniCS`
Solver : conic optimization `Mosek`

MBB beam

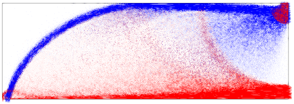


(a) L_1-Rankine

(b) Rankine

(c) von Mises

LOAD-MAX with $\eta = 0.20$, without penalisation



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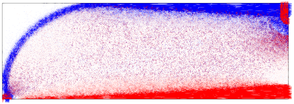
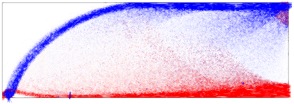


(b) Rankine



(c) von Mises

Principal stresses (compression/traction)

Alternative formulation: volume minimisation

Objective : Find $\Omega \subseteq \mathcal{D}$ with **minimum volume** for a given load level λ:

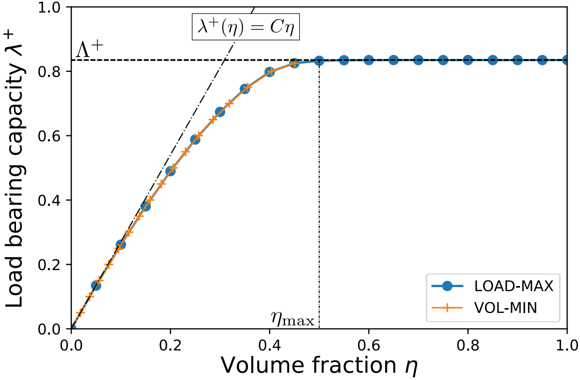
$$\begin{aligned} \eta^-(\lambda) = \min_{\rho,\boldsymbol{\sigma}} \quad & \frac{1}{|\mathcal{D}|}\int_{\mathcal{D}} \rho \,\mathrm{d}\Omega \\ \text{s.t.} \quad & \operatorname{div}\boldsymbol{\sigma} = 0 && \text{in } \mathcal{D} \\ & \boldsymbol{\sigma}\boldsymbol{n} = \lambda T && \text{in } \partial\mathcal{D}_N \\ & g_G(\boldsymbol{\sigma}) \leq \rho && \text{in } \mathcal{D} \\ & \rho \in [0;1] \end{aligned} \qquad \text{(VOL-MIN)}$$

can be more relevant for engineering practice

MBB beam

Results

- (VOL-MIN) = (LOAD-MAX)
- up to 50% of material savings with respect to the limit load



MBB beam (after penalization)

$p_{\max} = 3$, 20 iterations

(a) L_1-Rankine

(b) Rankine

(c) von Mises

penalized (`LOAD-MAX`)

(a) L_1-Rankine

(b) Rankine

(c) von Mises

penalized (`VOL-MIN`)

MBB beam (after penalization)

(a) $\ell = \dfrac{1}{24}h,\ \lambda^+ = 0.36\Lambda^+$

(b) $\ell = \dfrac{1}{12}h,\ \lambda^+ = 0.29\Lambda^+$

(c) $\ell = \dfrac{1}{6}h,\ \lambda^+ = 0.19\Lambda^+$

Gradient-control parameter ℓ

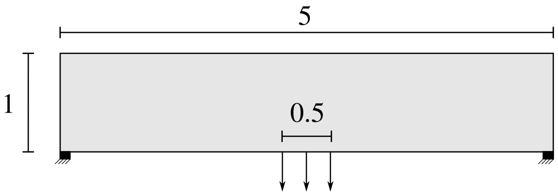
(a) $\eta = 0.2,\ \lambda^+ = 0.36\Lambda^+$

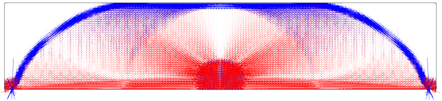
(b) $\eta = 0.3,\ \lambda^+ = 0.55\Lambda^+$

(c) $\eta = 0.4,\ \lambda^+ = 0.77\Lambda^+$

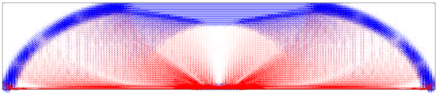
Volume constraint η

Material with asymmetric strengths





(a) $f_c/f_t = 10$

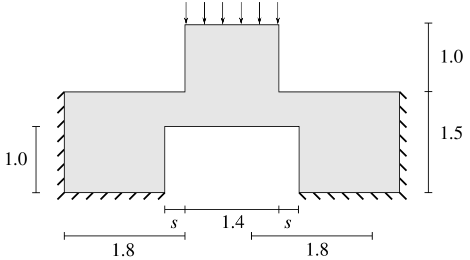


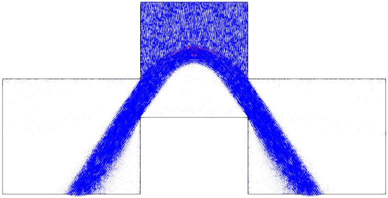
(b) $f_c/f_t = 0.1$

Principal stresses (compression/traction)

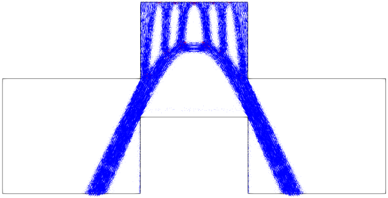
Material without tensile strength

ex: rock or masonry structures





(a) Before penalization

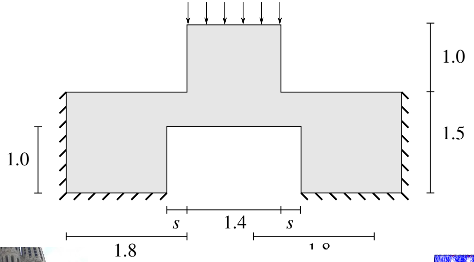


(b) After penalization

$s = 0$

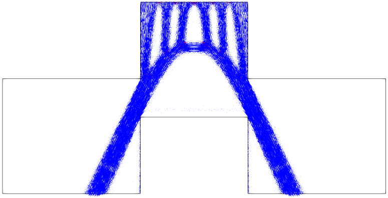
Material without tensile strength

ex: rock or masonry structures



(a) The Passion Façade

$s = 0$

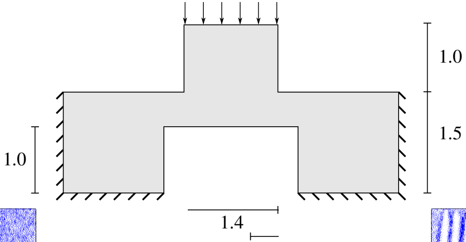


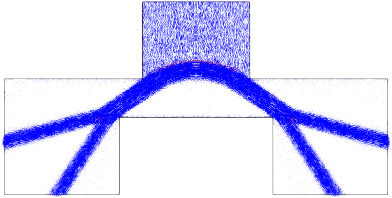
(b) After penalization

arch

Material without tensile strength

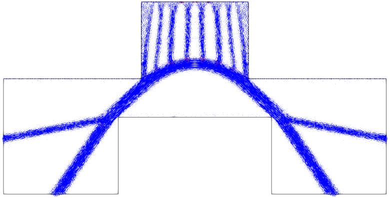
ex: rock or masonry structures





(a) Before penalization

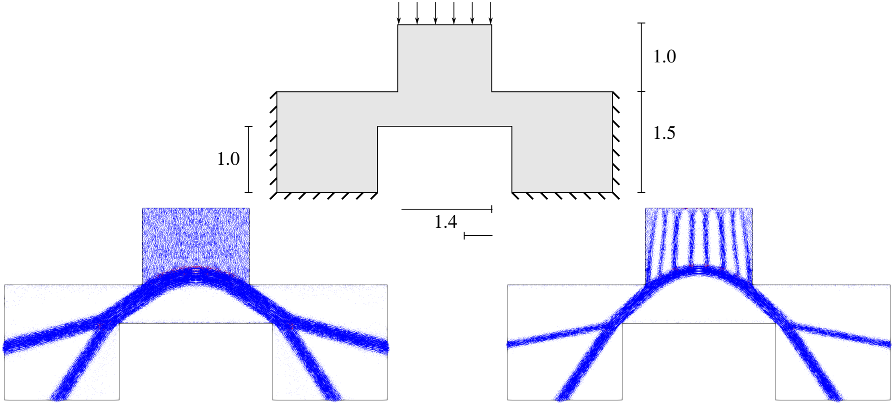
$s = 0.3$



(b) After penalization

Material without tensile strength

ex: rock or masonry structures

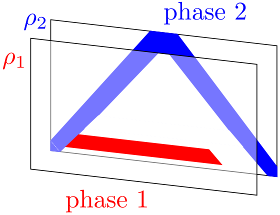


(a) Before penalization $s = 0.3$ **(b)** After penalization

arch-buttress

Outline

Topology optimization with two phases

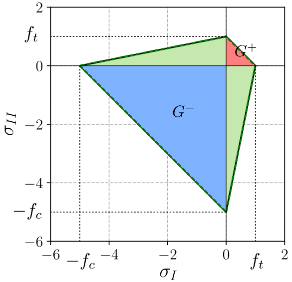


we want to optimize independently **two phases** (+ void)
e.g. steel and concrete, tension and compression

Strength condition

$$\boldsymbol{\sigma} \in G(\rho_1, \rho_2) \Leftrightarrow \exists \boldsymbol{\sigma}^1, \boldsymbol{\sigma}^2 \text{ s.t. } \begin{cases} \boldsymbol{\sigma} = \boldsymbol{\sigma}^1 + \boldsymbol{\sigma}^2 \\ \boldsymbol{\sigma}^1 \in \rho_1 G^1 \\ \boldsymbol{\sigma}^2 \in \rho_2 G^2 \end{cases}$$

with $\rho_1 + \rho_2 \leq 1$



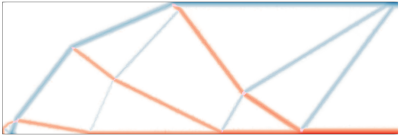
Problem formulation

$$
\begin{aligned}
\lambda^+ = \quad & \max_{\lambda,\boldsymbol{\sigma^1},\boldsymbol{\sigma^2},\rho_1,\rho_2} \lambda \\
\text{s.t.} \quad & \operatorname{div}(\boldsymbol{\sigma}^1+\boldsymbol{\sigma}^2) = 0 \\
& (\boldsymbol{\sigma}^1+\boldsymbol{\sigma}^2)\boldsymbol{n} = \lambda\boldsymbol{T} \\
& \boldsymbol{\sigma}^1 \in \rho_1 G^1 \\
& \boldsymbol{\sigma}^2 \in \rho_2 G^2 \\
& \int_{\mathcal{D}}(\rho_1+\rho_2)\,\mathrm{d}\Omega \leq \eta|\mathcal{D}| \\
& 0 \leq \rho_1 \leq 1 \\
& 0 \leq \rho_2 \leq 1 \\
& \rho_1+\rho_2 \leq 1
\end{aligned}
\qquad (\texttt{BIMAT-LOAD-MAX})
$$

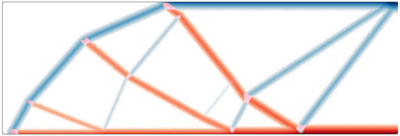
and similarly for the volume minimization

MBB beam

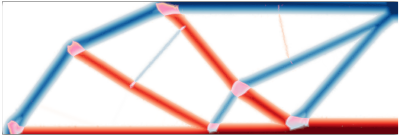
L_1-Rankine with **pure compression** G^- and **pure tension** G^+



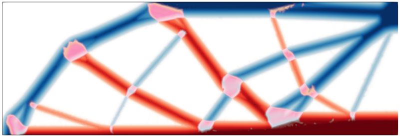
(a) $\eta = 0.05$



(b) $\eta = 0.1$



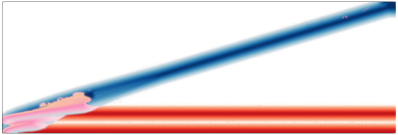
(c) $\eta = 0.2$



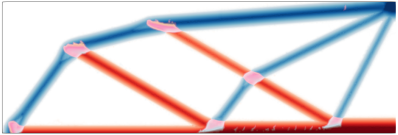
(d) $\eta = 0.3$

Poutre MBB

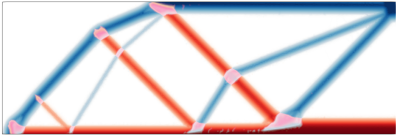
We can also enforce **specific orientations** for one phase e.g. the traction phase



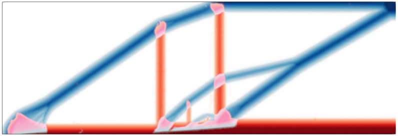
(a) Orientations along 0°



(b) Orientations along 0° and $\pm 30^\circ$



(c) Orientations along 0° and $\pm 45^\circ$



(d) Orientations along 0° and 90°

A deep-beam structure

The proposed methodology is linked with the use of **strut-and-tie models** for the design of reinforced-concrete structures

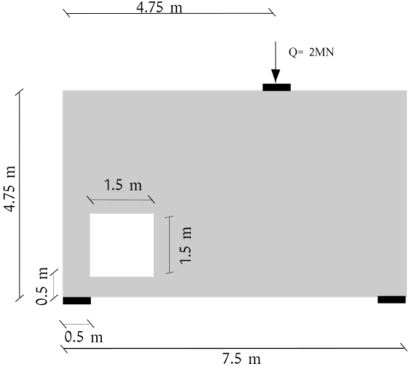


Figure – Deep beam with 25 cm width, analyzed in [Muttoni et al., 2015]

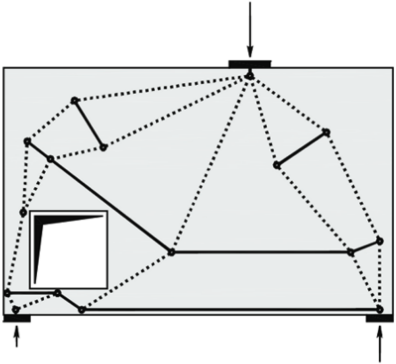


Figure – Strut-and-tie model proposed in [Muttoni et al., 2015]

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The proposed methodology is linked with the use of **strut-and-tie models** for the design of reinforced-concrete structures

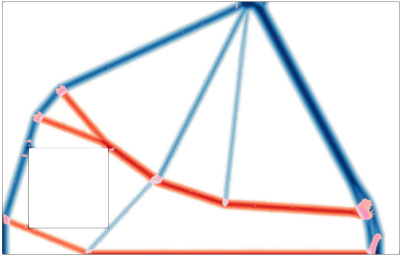

Figure – BIMAT-VOL-MIN

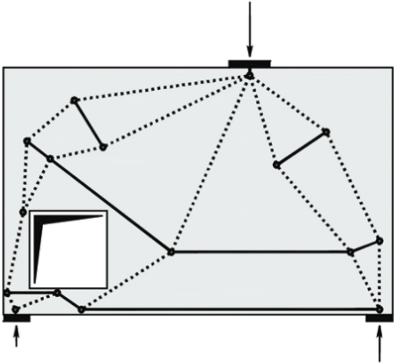

Figure – Strut-and-tie model proposed in [Muttoni et al., 2015]

A deep-beam structure

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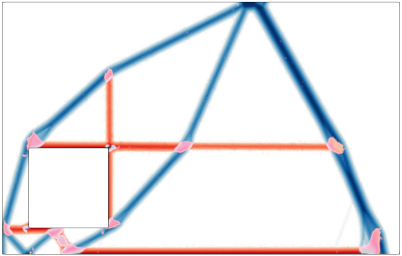


Figure – BIMAT-VOL-MIN, orthotropic

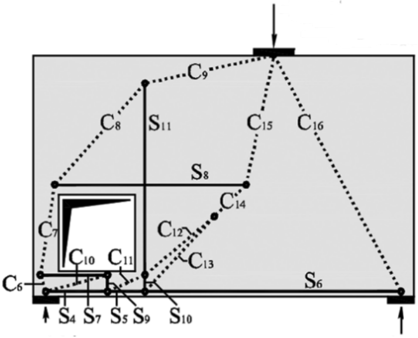
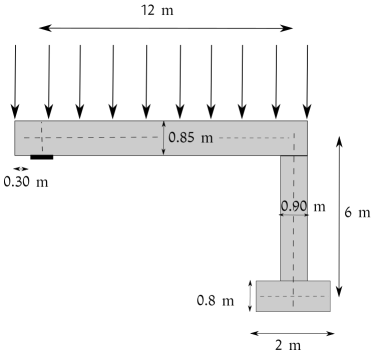
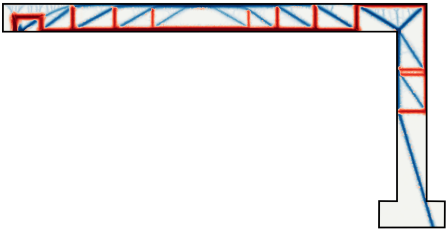


Figure – Strut-and-tie model with orthogonal reinforcements proposed in [Muttoni et al., 2015]

Beam and column





Conclusions & Perspectives

Conclusions

- extension of limit analysis concepts to topology optimization
- efficient numerical procedure
- liberty in strength criterion choice
- extension to bi-materials, automated strut-and-tie models

Mourad, L., Bleyer, J., Mesnil, R., Nseir, J., Sab, K., & Raphael, W. (2021). Topology optimization of load-bearing capacity. Structural and Multidisciplinary Optimization, 64(3), 1367-1383.

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Perspectives

- quantitative assessment
- specimen realization and testing
- 3D computations
- robust optimization wrt loading uncertainty

Conclusions & Perspectives

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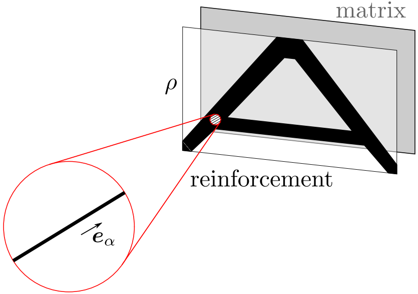
Thank you for your attention !

Anisotropic strength condition for reinforcements

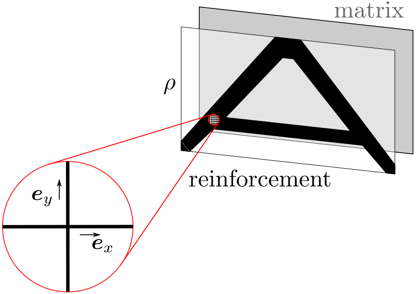


Uniaxial reinforcements

only one **known** direction $\boldsymbol{e}_\alpha$

$$G^{\mathrm{r}} = \{\sigma^{\mathrm{r}} \boldsymbol{e}_\alpha \otimes \boldsymbol{e}_\alpha \text{ s.t. } -f_c^{\mathrm{r}} \leq \sigma^{\mathrm{r}} \leq f_t^{\mathrm{r}}\}$$

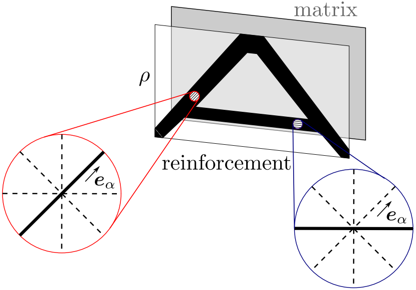
Anisotropic strength condition for reinforcements



Reinforcements along x and y

$$G^{\mathrm{r}} = \{\sigma^{\mathrm{r},x}\boldsymbol{e}_x \otimes \boldsymbol{e}_x + \sigma^{\mathrm{r},y}\boldsymbol{e}_y \otimes \boldsymbol{e}_y \\ \text{s.t. } -f_c^{\mathrm{r}} \leq \sigma^{\mathrm{r},x}, \sigma^{\mathrm{r},y} \leq f_t^{\mathrm{r}}\}$$

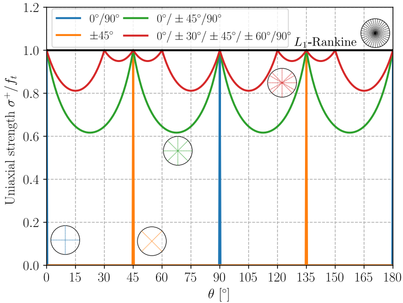
Anisotropic strength condition for reinforcements



Distributed uniaxial reinforcements

$$\boldsymbol{\sigma}^{\mathsf{r}} \in G^{\mathsf{r}} \Leftrightarrow \exists \sigma^{\mathsf{r},\alpha}, \zeta_\alpha \text{ s.t. } \begin{cases} \boldsymbol{\sigma}^{\mathsf{r}} = \displaystyle\sum_{\alpha \in \mathcal{A}} \zeta_\alpha \sigma^{\mathsf{r},\alpha} \boldsymbol{e}_\alpha \otimes \boldsymbol{e}_\alpha & \\ -f_c^{\mathsf{r}} \leq \sigma^{\mathsf{r},\alpha} \leq f_t^{\mathsf{r}} & \forall \alpha \in \mathcal{A} \\ \displaystyle\sum_{\alpha \in \mathcal{A}} \zeta_\alpha = 1, \quad \zeta_\alpha \in \{0;1\} & \forall \alpha \in \mathcal{A} \end{cases}$$

Anisotropic strength condition for reinforcements



Distributed uniaxial reinforcements (convexification)

$$\boldsymbol{\sigma}^{\mathsf{r}} \in G^{\mathsf{r}} \Leftrightarrow \exists \sigma^{\mathsf{r},\alpha}, \zeta_\alpha \text{ s.t. } \begin{cases} \boldsymbol{\sigma}^{\mathsf{r}} = \displaystyle\sum_{\alpha \in \mathcal{A}} \zeta_\alpha \sigma^{\mathsf{r},\alpha} \boldsymbol{e}_\alpha \otimes \boldsymbol{e}_\alpha & \\ -f_c^{\mathsf{r}} \leq \sigma^{\mathsf{r},\alpha} \leq f_t^{\mathsf{r}} & \forall \alpha \in \mathcal{A} \\ \displaystyle\sum_{\alpha \in \mathcal{A}} \zeta_\alpha = 1, \quad \zeta_\alpha \in [0;1] & \forall \alpha \in \mathcal{A} \end{cases}$$

L_1-**Rankine** = **isotropic** distribution ($\mathcal{A} = [0;\pi]$)